

# Open inflationary universes in the induced gravity theory

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(February 1, 2008)

The induced gravity theory is a variant of Jordan–Brans–Dicke theory where the ‘dilaton’ field possesses a potential. It has the unusual feature that in the presence of a false vacuum there is a *stable* static solution with the dilaton field displaced from the minimum of its potential, giving perfect de Sitter expansion. We demonstrate how this solution can be used to implement the open inflationary universe scenario. The necessary second phase of inflation after false vacuum decay by bubble nucleation is driven by the dilaton rolling from the static point to the minimum of its potential. Because the static solution is stable whilst the false vacuum persists, the required evolution occurs for a wide range of initial conditions. As the exterior of the bubble is perfect de Sitter space, there is no problem with fields rolling outside the bubble, as in one of the related models considered by Linde and Mezhlumian, and the expansion rates before and after tunnelling may be similar which prevents problematic high-amplitude super-curvature modes from being generated. Once normalized to the microwave background anisotropies seen by the COBE satellite, the viable models form a one-parameter family for each possible  $\Omega_0$ .

PACS numbers: 98.80.Cq

Sussex preprint SUSSEX-AST 96/7-7, astro-ph/9607166

## I. INTRODUCTION

Models of inflation leading to an open universe, which have a surprisingly long history [1–3], have received a new lease of life recently [4–6]. The idea relies on the observation that the inside of a bubble formed by quantum tunnelling looks exactly like an open universe to an observer inside the bubble [7], plus the realization that if a period of inflation occurs within the bubble after tunnelling it is possible to tune the present value of the density parameter inside the bubble to whatever value one desires, for example  $\Omega_0 \simeq 0.3$  as favoured by certain types of cosmological observation. An initial phase of inflation before tunnelling is also required, in order to provide a homogeneous background within which the bubble nucleation can occur.

The earliest specific realizations of the open inflation idea utilized a single scalar field with a rather complicated effective potential [3,5]. The potential requires a local minimum at non-zero energy, in which the trapped scalar field drives the first phase of inflation, plus a flat region of the potential on the other side of the barrier, down which the field can slowly roll after tunnelling to drive the second inflationary period. This second period must have a duration around 60  $e$ -foldings; much more and the universe will end up flat, much less and it will end up empty. While implementing the open inflation paradigm with just a single field is in principle an attractive possibility, the perceived unnaturalness of a potential with so many features mitigates against it.

A much more natural framework for open inflation was introduced in Ref. [6], where two scalar fields were used. In this case, the field which drives the second period of inflation is distinct from the field undergoing the tunnelling. The two fields need not even be explicitly cou-

pled — the potential

$$V(\phi, \sigma) = V(\sigma) + \frac{1}{2}m^2\phi^2 \quad (1)$$

can be used [6], where  $V(\sigma)$  is a potential featuring a metastable vacuum. The  $\phi$  field rolls down the potential from large  $|\phi|$  while  $\sigma$  is in the false vacuum driving the first period of inflation, then at some point  $\sigma$  tunnels generating the open universe, and then  $\phi$  continues to roll to the bottom of its potential driving the second phase of inflation as it goes. Linde and Mezhlumian [6] discuss several variants of this basic scheme.

Introducing the second field, while improving the naturalness of the models, can bring in problems of its own. The main one is that the space-time *outside* the bubble may no longer be perfectly de Sitter, due to evolution of the non-tunnelling field. This breaks the homogeneity at the bubble wall which forms the initial ( $t = 0$ ) hypersurface for the open universe within the bubble [6], even in the absence of bubble-wall fluctuations which we discuss later. In the model quoted above, this is a serious problem unless  $\phi$  can be made to roll much more slowly outside the bubble than inside. This can be achieved by ensuring that the energy density after tunnelling is much less than that before. However, even this solution may bring a new problem, discussed by Linde and Mezhlumian [6] and computed in detail by Sasaki and Tanaka [8], that long-wavelength modes, known as super-curvature modes, may be excessively generated if the difference in energy density before and after tunnelling is large. This problem is avoided in two other models in Ref. [6], where the fields are static before tunnelling.

In this paper we describe a new model for open inflation, based on the induced gravity theory [9]. In this theory, which is basically the Jordan–Brans–Dicke the-

ory [10,11] with a symmetry-breaking potential for the Brans–Dicke, or dilaton, field  $\phi$ , the field driving the second period of inflation is associated with the gravitational sector of the theory rather than introduced by hand into the matter sector. The tunnelling field, as in the models above, is to be associated with some symmetry breaking in the early universe.

This model is ideally suited for the implementation of open inflation, because in the presence of a false vacuum there is a stable, static solution for the dilaton where it is displaced from the minimum of its potential. All initial conditions evolve into this solution, which acts as a late-time attractor, as long as the false vacuum persists. It corresponds to a perfect de Sitter solution, and thus is an ideal environment for nucleation to take place. When the false vacuum decays, the dilaton suddenly ‘realizes’ that it is displaced from the minimum of its potential and slow-rolls into it, driving the necessary second period of inflation.

As we shall see, this scenario can be implemented for very reasonable choices of parameters. Once one normalizes the density perturbations produced by quantum fluctuations to the observations by the COBE satellite [12], one is left with a one-parameter family of possible models for each value of the present density parameter  $\Omega_0$ . Because there is no problem of rolling of the fields outside the bubble, there is no necessity for a large energy difference before and after tunnelling, and so no danger of excessive super-curvature modes.

## II. THE MODEL AND THE STATIC SOLUTION

The action for induced gravity is [9]

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \xi \phi^2 R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) + \mathcal{L}_{\text{mat}} \right], \quad (2)$$

where we shall refer to the field  $\phi$  as the dilaton. In our conventions, an Einstein–Hilbert term requires a positive sign, so  $\xi$  must be positive and the effective gravitational coupling is  $G_{\text{eff}} = 1/(8\pi\xi\phi^2)$ . The choice of potential for  $\phi$  is not very important for our purposes; we choose the simplest symmetry breaking form [9]

$$V(\phi) = \frac{1}{8} \lambda (\phi^2 - \nu^2)^2. \quad (3)$$

In order that the correct present strength of gravity is obtained when the  $\phi$  field sits in its minimum, we require

$$\nu^2 = m_{\text{Pl}}^2 / 8\pi\xi. \quad (4)$$

This action is in fact exactly that of Jordan–Brans–Dicke theory [10,11] with a potential for the Brans–Dicke field; the correspondence is given by, in the usual notation

$$\Phi = 8\pi\phi^2, \quad \omega = \frac{1}{4\xi}. \quad (5)$$

Because of the presence of the potential, the usual solar system and nucleosynthesis limits on  $\omega$  do not apply.

The matter lagrangian  $\mathcal{L}_{\text{mat}}$  contains all the other matter in the theory. We shall assume that during the initial inflationary phase this is dominated by a single scalar field  $\sigma$

$$\mathcal{L}_{\text{mat}} = -\frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + V(\sigma), \quad (6)$$

whose potential possesses both a metastable vacuum state and a true minimum. The detailed form of the potential is irrelevant; all we need to know is the energy of the false vacuum,  $V_{\text{fv}}$ , and the tunnelling rate  $\Gamma$  from this state to the true vacuum.

Previously, inflation during induced gravity symmetry breaking has mainly been considered with only one scalar field, the dilaton, and hence a single period of inflation [13–15]. Accetta and Trester [16], however, have considered the action above in the context of extended inflation, the essential difference being that they require percolation of the true vacuum bubbles whereas we require that the bubble nucleation rate is sufficiently small that the bubbles remain isolated.

The Friedmann equation and the equation of motion for  $\phi$  are

$$H^2 + 2H \frac{\dot{\phi}}{\phi} + \frac{k}{a^2} = \frac{1}{3\xi\phi^2} \left[ \frac{\dot{\phi}^2}{2} + V(\phi) + V(\sigma) \right], \quad (7)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\dot{\phi}^2}{\phi} = \frac{1}{1+6\xi} \left[ \frac{4(V(\phi) + V(\sigma))}{\phi} - \frac{dV}{d\phi} \right]. \quad (8)$$

Remarkably, while the  $\sigma$  field is in the false vacuum these equations possess an exact *static* solution for a particular value of  $\phi^2$ , driving a de Sitter expansion. To our knowledge, this solution has not appeared before in the literature.\* This occurs when the right hand side of Eq. (8) equals zero; labelling the appropriate value of  $\phi$  as  $\phi_{\text{st}}$ , it is given by

$$\phi_{\text{st}}^2 = \nu^2 \left( 1 + \frac{8V_{\text{fv}}}{\lambda\nu^4} \right), \quad (9)$$

leading to (at late times once the curvature term becomes negligible) the expansion rate

$$H_{\text{st}}^2 = \frac{8\pi V_{\text{fv}}}{3m_{\text{Pl}}^2}. \quad (10)$$

Interestingly, this looks exactly like the usual Friedmann equation, but note that it is the *present* Planck

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\* Accetta and Trester [16] considered exactly this action in a different context and displayed a solution of this type, but they appear to have derived it incorrectly (their Eq. (5) is missing a term).

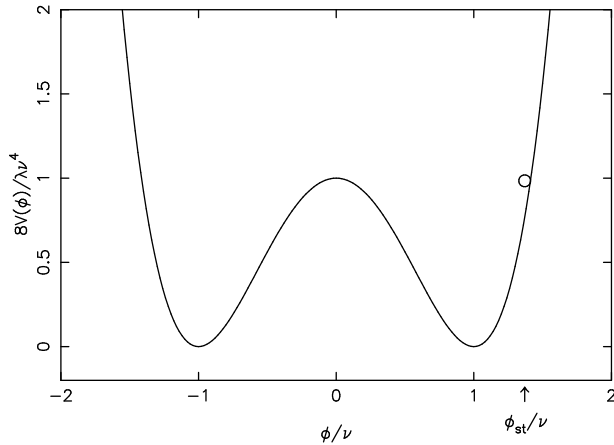


FIG. 1. The potential for the dilaton, with minima at  $\phi^2 = \nu^2$ . The location of the static point  $\phi_{\text{st}}$ , here taken to be positive and indicated by the circle, depends on the size of the false vacuum. When the false vacuum decays,  $\phi$  rolls to its true minimum, driving a second period of inflation.

mass which appears in this formula and not the effective Planck mass at that time, given by  $\sqrt{8\pi\xi\phi_{\text{st}}^2}$ . Note also that this solution only exists when the false vacuum is present; if  $V_{\text{fv}} = 0$  then the only static solution is  $\phi_{\text{st}} = \nu$  corresponding to a completely empty universe.<sup>†</sup>

An important feature of this solution is that it is stable. This is intuitively obvious (there being no other type of solution into which it can decay); we have also checked this explicitly by considering a linear perturbation about  $\phi_{\text{st}}$ . The static solution is stable regardless of parameters, but the nature of the stability can change; defining  $V_{\text{fv}}^{\text{crit}} = 4\xi\lambda\nu^4/3(1+6\xi)$ , then for  $V_{\text{fv}} < V_{\text{fv}}^{\text{crit}}$  the perturbation oscillates with exponentially decaying amplitude, while for  $V_{\text{fv}} > V_{\text{fv}}^{\text{crit}}$  it just decays exponentially. An alternative view of the stability is given below.

In fact, as long as the false vacuum persists the static solution is a late-time attractor for all initial conditions. For instance, even if  $\phi$  is placed in its minimum at  $\phi = \nu$ , it will then drift up the potential until it reaches the static solution. Therefore all regions of space in which  $\sigma$  is trapped for sufficiently long in its false vacuum reach the same physical state at late times. Fig. 1 illustrates this behaviour schematically.

The static solution displaced from the minimum of the potential seems unusual in the theory as written, but this is simply because there are extra terms, acting similarly to an effective potential, coming from the coupling to gravity, so that  $V(\phi)$  is insufficient to determine the dynamics of the dilaton. An alternative viewpoint of the static solution comes from considering instead the con-

<sup>†</sup>However, other contributions to  $\mathcal{L}_{\text{mat}}$ , such as non-relativistic matter, can also displace  $\phi$  from its minimum at later epochs.

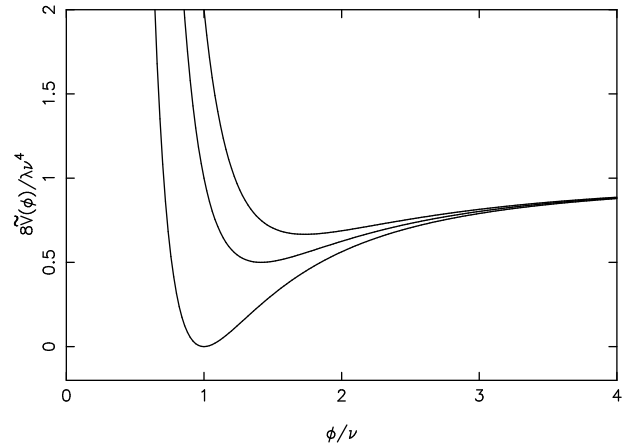


FIG. 2. The potential  $\tilde{V}(\phi)$  of the dilaton in the Einstein frame, shown in normalized units for different choices of the false vacuum energy — from bottom to top  $8V_{\text{fv}}/\lambda\nu^4 = 0, 1$  and  $2$ . The static point is at the minimum of  $\tilde{V}(\phi)$ , and is displaced to the right of  $\phi = \nu$  by the presence of the false vacuum.

formally related Einstein frame, where the coupling of the dilaton to gravity is removed by the appropriate conformal transformation. We make the transformation

$$dt = C^{-1}d\tilde{t} \quad , \quad a(t) = C^{-1}\tilde{a}(\tilde{t}) \quad , \quad (11)$$

where the required conformal factor is  $C = \phi/\nu$ . In order to have a canonical kinetic term, we can redefine the dilaton according to

$$\frac{\tilde{\phi}}{m_{\text{Pl}}} = \sqrt{\frac{1+6\xi}{32\pi\xi}} \ln\left(\frac{\phi^2}{\nu^2}\right) \quad . \quad (12)$$

In the Einstein frame, the potential for the dilaton is given by

$$\frac{\tilde{V}(\phi)}{\nu^4} = \frac{\lambda(\phi^2 - \nu^2)^2}{8\phi^4} + \frac{V_{\text{fv}}}{\phi^4} \quad , \quad (13)$$

where we have left things in terms of the original  $\phi$  field. The minimum of this potential is indeed at  $\phi_{\text{st}}$ , and this is the only local minimum of the potential and hence the late-time attractor for all initial conditions, provided the false vacuum persists. The nature of the stability as given above can also be derived in this framework. Fig. 2 shows  $\tilde{V}(\phi)$  for a selection of false vacuum energies.

### III. INFLATION AFTER TUNNELLING

#### A. Creating an open universe

The static solution provides the initial environment in which open inflation can be realized, by providing a large homogeneous region. An open universe is now created by

tunnelling of the  $\sigma$  field to its true vacuum, leaving only the gravitational sector of the theory. With  $V_{\text{fv}}$  now gone, the static point loses its support and the  $\phi$  field ‘notices’ that it is displaced from its true minimum. It therefore begins to evolve towards it. Because  $V(\phi)$  is quite a flat potential, inflation can proceed while it does so. The interesting situation is where just enough inflation occurs inside the bubble so as to generate an open, but not too empty, universe at the present. The precise amount of inflation required to do this depends somewhat on the energy scale of inflation and on the details of reheating; for definiteness we shall simply assume that 60  $e$ -foldings of inflation in the second epoch are sufficient to place the present density parameter  $\Omega_0$  somewhere in the interesting range  $0.1 < \Omega_0 < 0.9$  [5].

Inflation in the induced gravity theory without a false vacuum, now essentially just a single field model, has already been investigated [13–15]. We simply need to locate the point on the potential corresponding to 60  $e$ -foldings from the end,  $\phi_{60}$ , and ensure that it corresponds to the initial condition for the second epoch. That is, we arrange that  $\phi_{\text{st}} = \phi_{60}$ , by making the appropriate choice of the false vacuum energy  $V_{\text{fv}}$  in Eq. (9). A displacement of  $V_{\text{fv}}$  from this criterion alters the initial condition for the second phase, and hence results in a different  $\Omega_0$ . Any value of the present density parameter can therefore be arranged by making the appropriate choice of false vacuum energy.

Notice that in this model, unlike those discussed in Ref. [6], every bubble which nucleates gives rise to the same value of  $\Omega_0$ , assuming that the parameters of the underlying theory are fixed and that the region of the universe nucleated from the static solution. In Ref. [6], models were discussed in which different bubbles could have different density parameters, giving an ensemble of different density universes.

### B. Avoiding bubble percolation

In order to obtain a homogeneous open universe we require that the bubbles of true vacuum produced are isolated; we don’t want our bubble universe to have already collided with others. The nucleation rate per Hubble volume per Hubble time is given by  $E = \Gamma/H^4$ , where  $\Gamma$  is the tunnelling rate per unit volume per unit time determined by the shape of the potential barrier between the false and true vacuum states. Provided  $E$  is some way less than one, bubbles do not percolate and the phase transition is unable to complete — the graceful exit problem of old inflation. If  $E$  is much less than one, then any bubble collisions at all are extremely rare, and a single-bubble universe can survive in isolation within the surrounding ‘sea’ of de Sitter space. Gott and Statler [2] found that  $E < 1/400$  was sufficient to keep the bubbles isolated, for any  $\Omega_0 \geq 0.2$ .

Since  $\Gamma$  is otherwise unconstrained, and since tun-

nelling rates commonly are strongly exponentially suppressed, there is no problem in us choosing a nucleation rate so low as to keep our scenario viable. The minimum value of  $H$  of relevance will be that of the static point, and keeping  $E < 1/400$  only requires

$$\Gamma < \frac{4\pi^2 V_{\text{fv}}^2}{225 m_{\text{Pl}}^4}. \quad (14)$$

### C. Dynamics of the second inflationary phase

Once  $\sigma$  has tunnelled to the minimum of its potential, nucleating a bubble of true vacuum  $V(\sigma) = 0$ , the  $\phi$  field then rolls towards the minimum of its potential at  $\phi^2 = \nu^2$  as in single-field induced gravity inflation. We will take  $\phi$  to be positive, and make the slow-roll approximations

$$\dot{\phi}^2 \ll V, \quad \frac{\dot{\phi}^2}{\phi} \ll 3H\dot{\phi}, \quad \ddot{\phi} \ll 3H\dot{\phi}. \quad (15)$$

Dropping the curvature term, which becomes negligible once inflation has commenced, the equations of motion have the simple solution [15]

$$\dot{\phi} = -\frac{m_{\text{Pl}}^2}{8\pi} \frac{1}{1+6\xi} \sqrt{\frac{2\lambda}{3\xi}}, \quad (16)$$

$$H = \sqrt{\frac{\lambda}{24\xi}} \left(1 - \frac{m_{\text{Pl}}^2}{8\pi\xi\phi^2}\right) \phi. \quad (17)$$

Meanwhile, outside the bubble  $\phi$  remains at the static point with  $H$  constant, so that the motion of the  $\sigma$  and  $\phi$  fields is synchronized and the initial hypersurface for the open universe, the bubble wall, remains homogeneous.

Taking  $\ddot{a} > 0$ , applied to the slow-roll solution, as the condition for inflation to occur, inflation ceases once  $\phi = \phi_{\text{end}}$ , where

$$\phi_{\text{end}}^2 = \frac{1+8\xi+2\sqrt{\xi(2+13\xi)}}{1+6\xi} \nu^2. \quad (18)$$

So inflation ends slightly before  $\phi$  reaches the minimum at  $\phi = \nu$ , though the difference is not important.

In the slow-roll limit the amount of inflation measured in different conformal frames is the same [17], so the standard expression for the number of  $e$ -foldings

$$N = \int_{\phi_{\text{end}}}^{\phi} \frac{H}{\dot{\phi}} d\phi, \quad (19)$$

can be applied directly. Substituting in the slow-roll solution, then to have 60  $e$ -foldings we require

$$\frac{\phi_{\text{st}}^2}{m_{\text{Pl}}^2} = \frac{\phi_{\text{end}}^2}{m_{\text{Pl}}^2} + \frac{1}{4\pi\xi} \ln \frac{\phi_{\text{st}}}{\phi_{\text{end}}} + \frac{60}{\pi(1+6\xi)}. \quad (20)$$

Accetta, Zoller and Turner [13] found approximate analytic solutions for  $\phi(N)$  in each of the limits  $\xi \ll 1/240$  and  $\xi \gg 1/240$ , with the additional approximation that  $\phi_{\text{end}} = \nu$ . However we will not use this approach as values of  $\xi$  between these limits are also interesting; instead we solve Eq. (20) numerically by iteration.

The false vacuum energy of the  $\sigma$  field can be calculated from the condition that there are 60  $e$ -foldings of inflation in the second phase, giving

$$V_{\text{fv}} = \frac{\lambda m_{\text{Pl}}^4}{64\pi\xi} \left( \frac{\phi_{\text{st}}^2}{m_{\text{Pl}}^2} - \frac{1}{8\pi\xi} \right). \quad (21)$$

#### IV. DENSITY PERTURBATIONS

There are three recognized types of density perturbations in open inflation models [18], sub-curvature modes, super-curvature modes and modes associated with bubble wall fluctuations.

##### A. Sub-curvature modes

Except on the largest scales, perturbations can be considered to have formed during the second phase of inflation. The standard calculation requires Einstein gravity and a canonically normalized scalar field, which is brought about by the conformal transformation Eq. (11) and field redefinition Eq. (12). Then the standard formula for the density perturbations can be applied, namely [19]

$$\delta_{\text{H}} = \frac{1}{5\pi} \frac{\tilde{H}^2}{|d\tilde{\phi}/d\tilde{t}|}, \quad (22)$$

where tildes refer to quantities in the Einstein frame. The quantities on the right-hand side should be evaluated when the relevant scales crossed outside the Hubble radius during inflation. The Einstein frame quantities can be related to those in the original frame without explicitly performing the conformal transformation, by using

$$\tilde{H} = \frac{1}{C}H + \frac{1}{C^2} \frac{dC}{dt}, \quad (23)$$

$$\frac{d\tilde{\phi}}{d\tilde{t}} = \frac{m_{\text{Pl}}^2}{8\pi\phi^2} \frac{\sqrt{1+6\xi}}{\xi} \frac{d\phi}{dt}. \quad (24)$$

Employing the slow-roll approximation, which allows us to drop the second term in the expression for  $\tilde{H}$ , we obtain

$$\delta_{\text{H}} = \sqrt{\frac{\lambda}{150}} \sqrt{\frac{1+6\xi}{\xi}} \frac{\phi^2}{m_{\text{Pl}}^2} \left( 1 - \frac{m_{\text{Pl}}^2}{8\pi\xi\phi^2} \right)^2. \quad (25)$$

The microwave anisotropies seen by COBE [12] correspond to perturbations generated about 60  $e$ -foldings

from the end of inflation, obtained by substituting  $\phi \simeq \phi_{\text{st}}$  in Eq. (25). In a spatially flat universe, the observed amplitude is reproduced if  $\delta_{\text{H}} \simeq 2 \times 10^{-5}$  [20]. The required value actually changes somewhat if  $\Omega_0$  is significantly less than one [20], but at the level of accuracy we are working we can ignore this correction. Obtaining the correct density perturbations determines, for a given  $\xi$ , the value of  $\lambda$  required.

##### B. Super-curvature and bubble wall modes

In models where the mass in the false vacuum obeys  $m^2 < 2H^2$ , one typically expects a discrete super-curvature mode [21,18]. The potential for  $\sigma$  is unconstrained in this respect, and so may or may not support such a mode. Much more important is the dilaton; considering the Einstein frame quantities, and denoting the ratio by  $\mu$ , we have

$$\mu^2 \equiv \frac{m_{\phi}^2}{\tilde{H}^2} \Big|_{\phi=\phi_{\text{st}}} = \frac{3\xi}{1+6\xi} \frac{\lambda\nu^4}{V_{\text{fv}}}, \quad (26)$$

which is typically small and so a super-curvature mode from the  $\phi$  field is expected.<sup>‡</sup>

For computing this mode, our situation is extremely similar to the ‘supernatural’ model of Linde and Mezhlumian [6]; the fields are initially both trapped, and then one becomes free to roll after tunnelling. The full spectrum of modes in the supernatural model was computed by Yamamoto et al. [18], in the limit where the expansion rate before and after tunnelling is taken to be the same. As we shall see, we are typically in this limit. They find that while there is a contribution from the super-curvature mode, it is never very large regardless of  $\mu$  and so does not threaten the viability of that model. A detailed calculation for our model is outside the scope of this paper, but we expect that the result is extremely similar, so long as the assumption that the expansion rate before and after tunnelling is equal holds.

Sasaki and Tanaka [8] have made detailed calculations of the situation where the expansion rate changes significantly during tunnelling, with particular reference to the model of Eq. (1). They found that the super-curvature fluctuations are enhanced, relative to the sub-curvature ones, by a factor of order  $(H_{\text{false}}/H_{\text{true}})^2$ , where these are the expansion rates before and after tunnelling. Such an enhancement could distort the cosmic microwave background (CMB) spectrum from that observed, since the super-curvature modes only contribute significantly to anisotropies with multipole number  $l \lesssim 10$  [22].

In our model this factor is given by

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<sup>‡</sup>We thank Juan García-Bellido for pointing this out to us.

$$\left(\frac{H_{\text{false}}}{H_{\text{true}}}\right)^2 = \frac{\lambda m_{\text{Pl}}^2}{8\xi V_{\text{fv}}} \frac{(\phi^2 - \nu^2)^2}{\phi^2}. \quad (27)$$

Evaluating this expression immediately after tunnelling gives

$$\left(\frac{H_{\text{false}}}{H_{\text{true}}}\right)^2 = 1 + \frac{\lambda \nu^4}{8V_{\text{fv}}}, \quad (28)$$

which leads to a limit on the ratio of the false vacuum energies of the  $\phi$  and  $\sigma$  fields.

Compared to the model of Eq. (1), we have the considerable advantage that the fields outside the bubble are already static, rather than having to be damped by a large difference in the expansion rate. As long as  $\xi$  is not too small, the ratio of energy densities is never big and so enhancement of the super-curvature modes will not be a problem for us. This is also the case with other models proposed in Ref. [6].

The final type of density perturbations are those associated with fluctuations in the bubble wall [23,18]. These will be no different from those in other types of open inflation model, where they have not proven constraining.

## V. PARAMETERS

For any particular value of  $\Omega_0$ , we can freely choose  $\xi$  but then the rest of the parameters of the model are uniquely determined:  $\nu$  from reproducing the present-day Planck mass,  $V_{\text{fv}}$  (in terms of  $\lambda$ ) from the required position of  $\phi_{\text{st}}$ , then  $\lambda$  from the magnitude of the density perturbations. Finally, an upper bound is placed on  $\Gamma$  from the requirement that the true-vacuum bubbles do not percolate.

In the limit  $\xi \ll 1/240$ , we have  $\phi_{\text{st}} \simeq \nu$  and can solve Eq. (20) by small parameter expansion. The parameters have approximate analytic forms

$$\lambda \approx 4.1 \times 10^{-11} \xi, \quad (29)$$

$$V_{\text{fv}} \approx 2.5 \times 10^{-13} \xi^{-1/2} m_{\text{Pl}}^4, \quad (30)$$

$$\Gamma < 1.1 \times 10^{-26} \xi^{-1} m_{\text{Pl}}^4, \quad (31)$$

$$\left(\frac{H_{\text{false}}}{H_{\text{true}}}\right)^2 \approx 1 + 0.03 \xi^{-1/2}, \quad (32)$$

$$\mu^2 = 0.8 \xi^{1/2}. \quad (33)$$

Away from that limit, we must solve Eq. (20) numerically. For a sample value of  $\xi = 10^{-2}$  we find

$$\lambda = 1.1 \times 10^{-12}, \quad (34)$$

$$V_{\text{fv}} = 1.4 \times 10^{-11} m_{\text{Pl}}^4, \quad (35)$$

$$\Gamma < 3.6 \times 10^{-23} m_{\text{Pl}}^4, \quad (36)$$

$$\left(\frac{H_{\text{false}}}{H_{\text{true}}}\right)^2 = 1.15, \quad (37)$$

$$\mu^2 = 0.03. \quad (38)$$

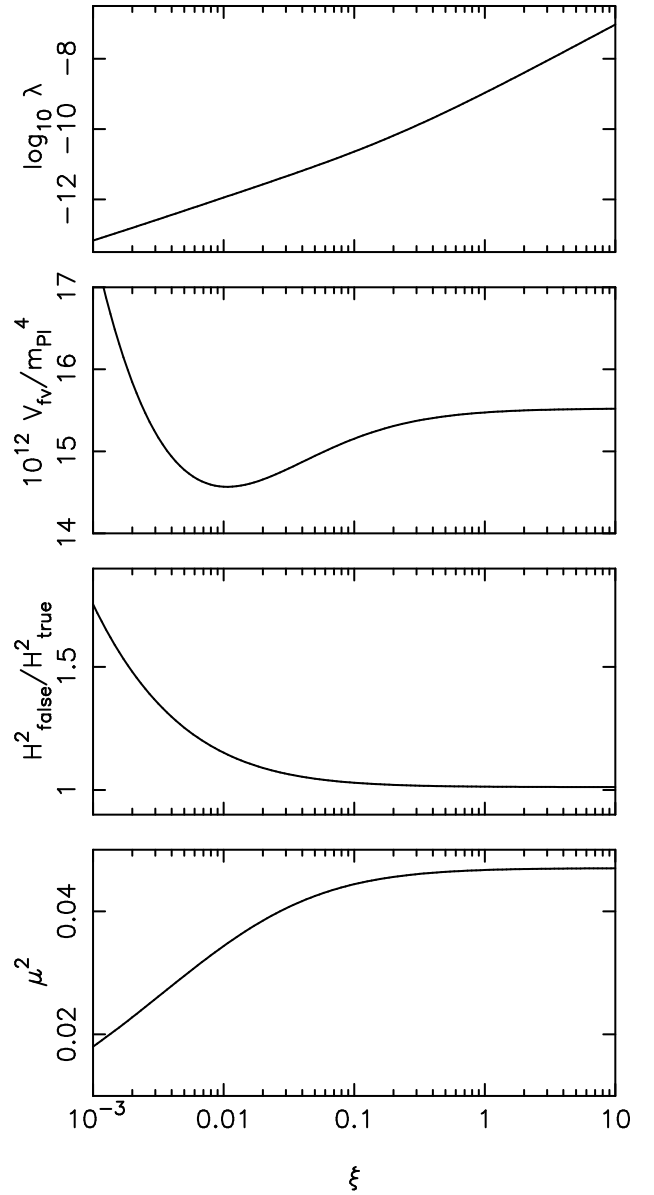


FIG. 3. The principal parameters as functions of  $\xi$ , for COBE-normalized models and assuming 60  $e$ -foldings in the second phase. For  $\xi \ll 1/240$ , the analytic forms given in the text apply.

In Fig. 3, we plot the values of the different parameters as functions of  $\xi$ , under the assumption of 60  $e$ -foldings in the second phase. The COBE normalization fixes the other quantities as functions of  $\xi$  alone once the number of  $e$ -foldings has been fixed. As we stated earlier, the precise value of  $\Omega_0$  this corresponds to depends on the reheating energy and other related uncertainties, but clearly the numbers will not change much if the number of  $e$ -foldings is varied somewhat to cover the entire interesting range of  $\Omega_0$  values.

Notice that these are all very reasonable values. The coupling  $\xi$  to gravity can easily be of order one or less, and the false vacuum energy is around the standard uni-

fication energy. Notice by how little  $V_{\text{fv}}$  changes as  $\xi$  is varied. The self-coupling  $\lambda$  does need to be small in order to give the correct magnitude of density perturbations, but this is a familiar requirement from a wide range of inflation models.

## VI. SUMMARY

We have constructed a new model of open inflation, based on the induced gravity theory, which capitalizes on the existence of a static, stable de Sitter solution in the presence of a false vacuum. We have shown that this model can easily satisfy all known constraints, both from inflationary dynamics and from the production of density perturbations. The value of  $\Omega_0$  is uniquely determined from the model parameters, though uncertainties related to reheating prevent an accurate calculation at present. It would be extremely interesting to see the results of a full calculation of all types of density perturbations in this model, along the lines of Ref. [18].

## ACKNOWLEDGMENTS

A. M. G. is supported by PPARC and A. R. L. by the Royal Society. We are indebted to Juan García-Bellido for a series of discussions on super-curvature modes, and also thank Jim Lidsey, Andrei Linde and David Wands for their comments. We acknowledge use of the Starlink computer system at the University of Sussex.

- [12] C. L. Bennett et al., *Astrophys. J. Lett.* **464**, L1 (1996).
- [13] F. S. Accetta, D. J. Zoller and M. S. Turner, *Phys. Rev. D* **31**, 3046 (1985).
- [14] D. I. Kaiser, *Phys. Rev. D* **49**, 6347 (1994).
- [15] R. Fakir and W. G. Unruh, *Phys. Rev. D* **41**, 1792 (1990).
- [16] F. S. Accetta and J. J. Trester, *Phys. Rev. D* **39**, 2854 (1989).
- [17] A. M. Green and A. R. Liddle, to appear, *Phys. Rev. D*, astro-ph/9604001 (1996).
- [18] K. Yamamoto, M. Sasaki and T. Tanaka, e-print archive: astro-ph/9605103 (1996).
- [19] A. R. Liddle and D. H. Lyth, *Phys. Rep.* **231**, 1 (1993).
- [20] E. F. Bunn and M. White, e-print archive: astro-ph/9607060 (1996).
- [21] M. Sasaki, T. Tanaka and K. Yamamoto, *Phys. Rev. D* **51**, 2979 (1995).
- [22] J. García-Bellido, A. R. Liddle, D. H. Lyth and D. Wands, *Phys. Rev. D* **52**, 6750 (1995).
- [23] J. García-Bellido, to appear, *Phys. Rev. D*, astro-ph/9510029 (1995); J. Garriga, to appear, *Phys. Rev. D*, gr-qc/9602025 (1996).

- 
- [1] J. R. Gott, *Nature* **295**, 304 (1982).
  - [2] J. R. Gott and T. S. Statler, *Phys. Lett.* **136B**, 157 (1984).
  - [3] J. R. Gott in *Inner Space, Outer Space*, eds. E. W. Kolb, M. S. Turner, D. Lindley, K. Olive and D. Seckel, University of Chicago Press (1986).
  - [4] M. Sasaki, T. Tanaka, K. Yamamoto and J. Yokoyama, *Phys. Lett. B* **317**, 510 (1993).
  - [5] M. Bucher, A. S. Goldhaber and N. Turok, *Phys. Rev. D* **52**, 3314; M. Bucher and N. Turok, *Phys. Rev. D* **52**, 5538 (1995).
  - [6] A. D. Linde, *Phys. Lett. B* **351**, 99 (1995); A. D. Linde and A. Mezhlumian, *Phys. Rev. D* **52**, 6789 (1995).
  - [7] S. Coleman and F. de Luccia, *Phys. Rev. D* **21**, 3305 (1980).
  - [8] M. Sasaki and T. Tanaka, e-print archive: astro-ph/9605104 (1996).
  - [9] A. Zee, *Phys. Rev. Lett.* **42**, 417 (1979).
  - [10] C. Brans and R. H. Dicke, *Phys. Rev.* **24**, 925 (1961).
  - [11] C. M. Will, *Theory and Experiment in Gravitational Physics*, Cambridge University Press (Cambridge, 1993).